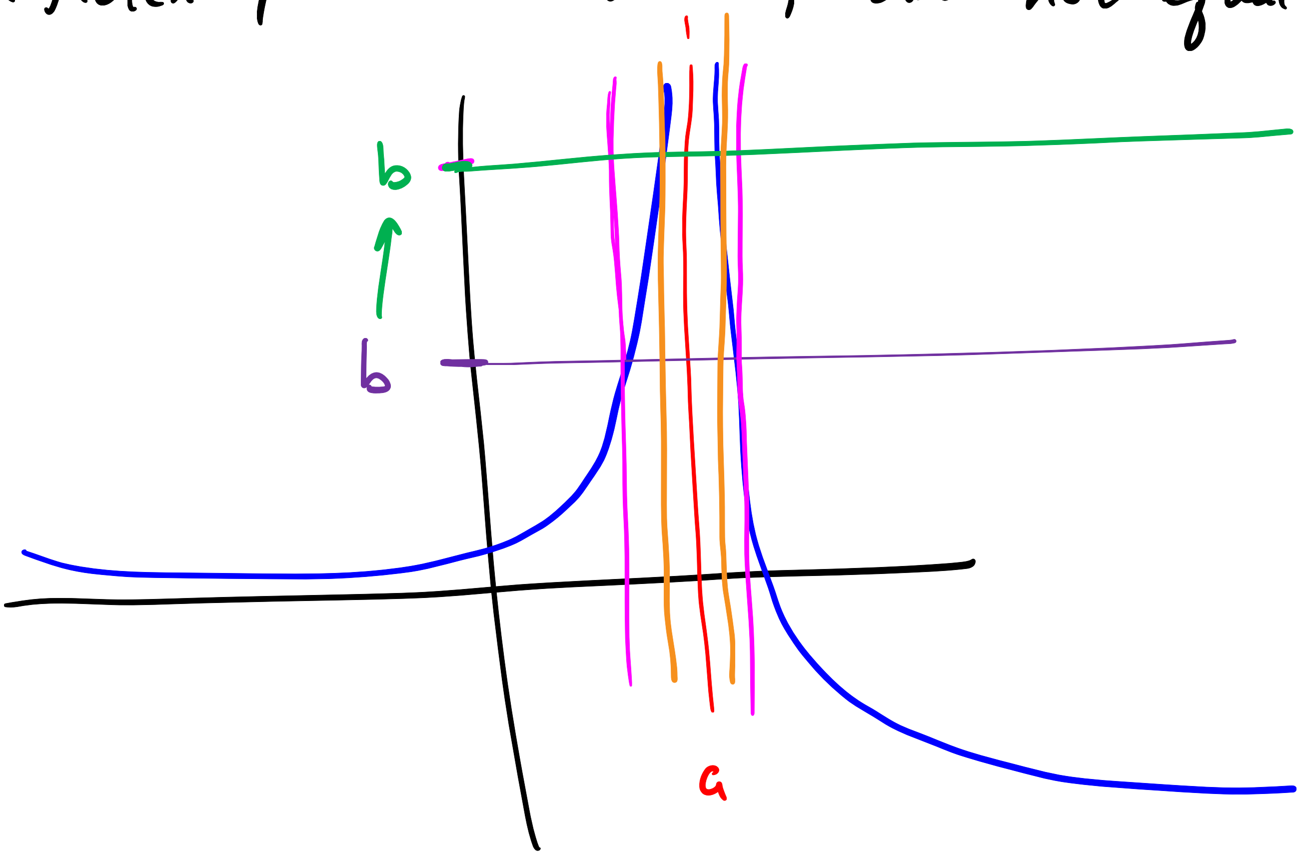
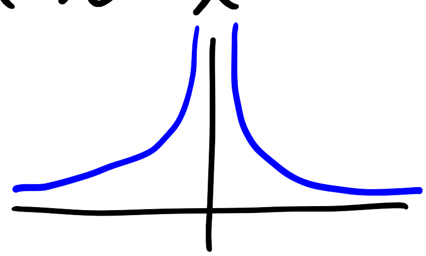
Def: Let f be a function defined on both sides $x=a$, except possibly at a. Then "$\lim_{x\to a} f(x)=\infty$" means that the values of $f(x)$ can be made arbitrarily large by taking x sufficiently close to a, but not equal to a.



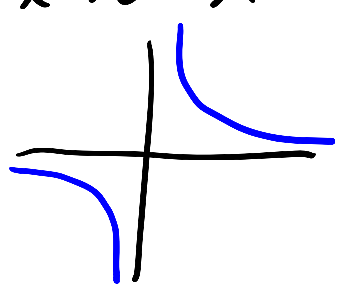
$$\lim_{x\to 0}\frac{1}{x^2}=\infty$$



$$\lim_{x\to 0}\frac{1}{x} \quad \text{DNE}$$

$$\lim_{x\to 0^-}\frac{1}{x}=-\infty$$

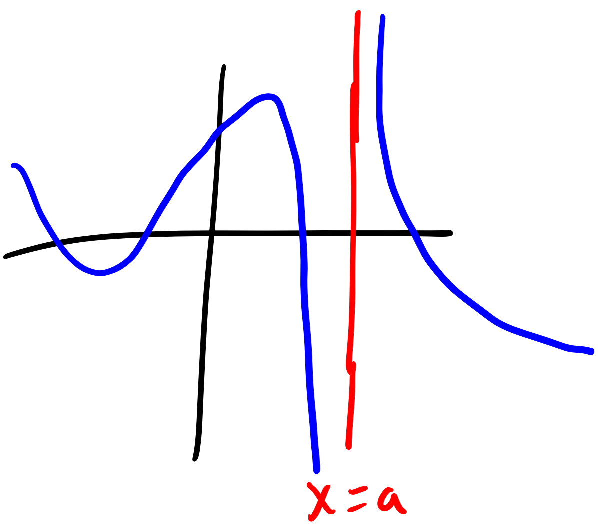
$$\lim_{x\to 0^+}\frac{1}{x}=\infty$$

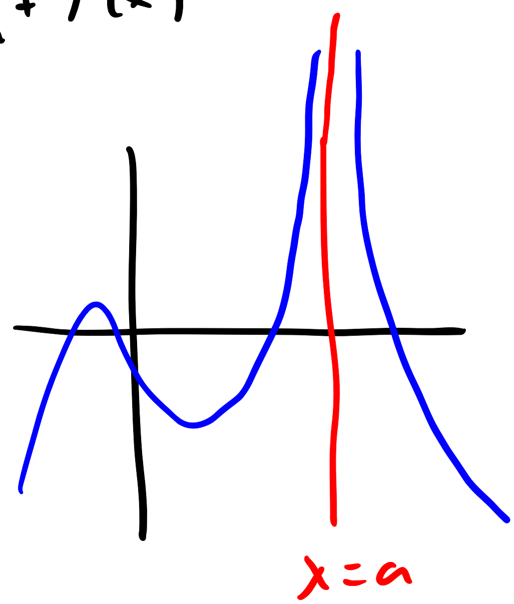


Def: The vertical line $x=a$ is a **vertical asymptote** of $y=f(x)$ if any of the following are true

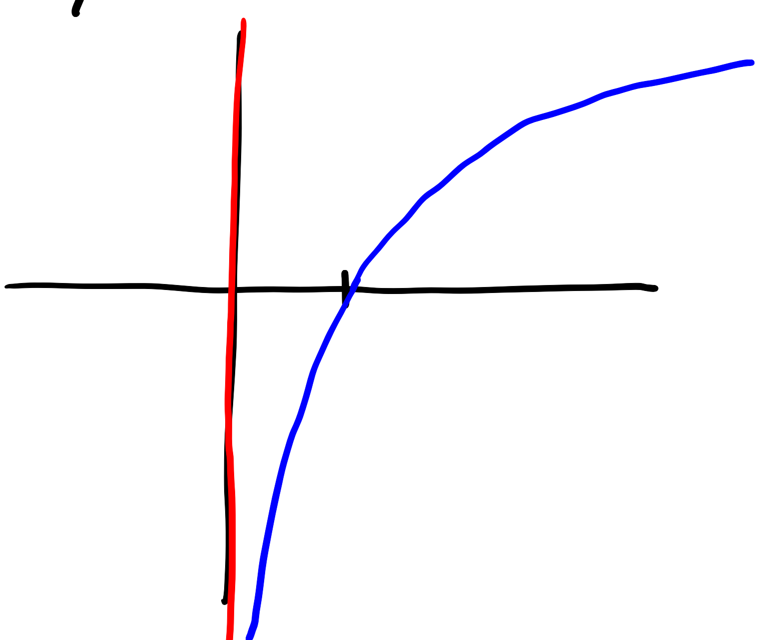
$$\lim_{x\to a} f(x) = \pm\infty, \quad \lim_{x\to a^-} f(x) = \pm\infty,$$

$$\text{or} \quad \lim_{x\to a^+} f(x) = \pm\infty$$



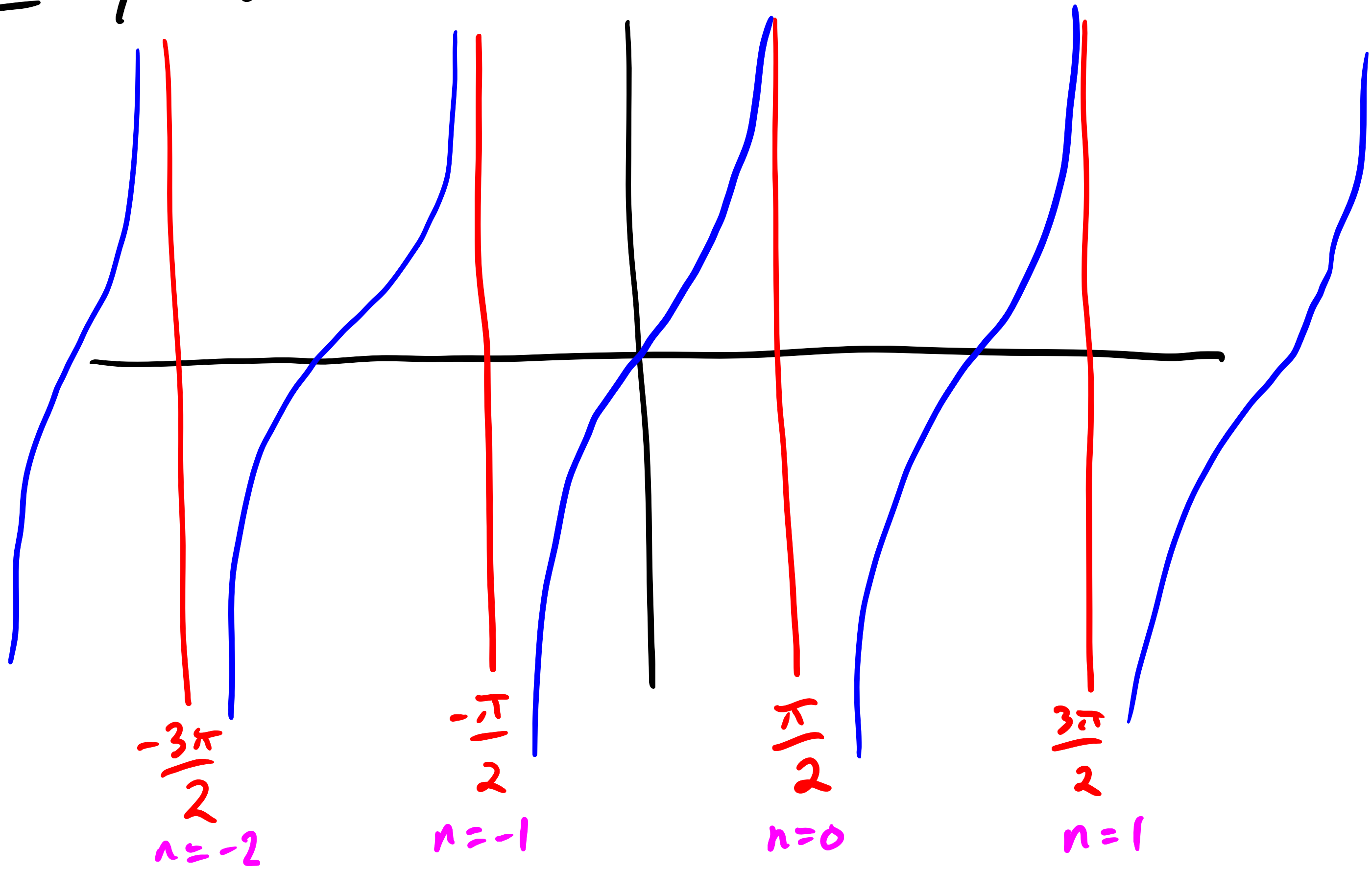


Ex $y = \ln x$



$$\lim_{x\to 0^+} \ln x = -\infty$$

Ex: $y = \tan x = \frac{\sin x}{\cos x}$



$$\lim_{x \to \left(\frac{(2n+1)\pi}{2}\right)^+} \tan x = -\infty$$

$$\lim_{x \to \left(\frac{(2n+1)\pi}{2}\right)^-} \tan x = \infty$$

Ex: $\lim_{x\to 3^+} \frac{x+1}{2x-6}$

* $x=3$ is a v.a. so the limit is $\pm\infty$

Choose $x=3.1$ & plug in

$$\frac{3.1+1}{2(3.1)-6} = \frac{4.1}{0.2} > 0$$

So, since this is positive, $\lim_{x\to 3^+} \frac{x+1}{2x-6} = \infty$.

Limit Laws (also applies to one-sided)

Suppose c is a constant and the limits

$$\lim_{x\to a} f(x) \quad \text{and} \quad \lim_{x\to a} g(x)$$

exist. Then

① $\lim_{x\to a} [f(x) \pm g(x)] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$

② $\lim_{x\to a} [c f(x)] = c \cdot \lim_{x\to a} f(x)$

③ $$\lim_{x\to a}[f(x)g(x)] = \left(\lim_{x\to a} f(x)\right)\left(\lim_{x\to a} g(x)\right)$$

④ $$\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$$ provided $\lim_{x\to a} g(x) \neq 0$

$$\left(\lim_{x\to 0}\frac{\sin x}{x} = 1 \qquad \begin{array}{l}\lim_{x\to 0}\sin x = 0\\ \lim_{x\to 0} x = 0\end{array}\right)$$

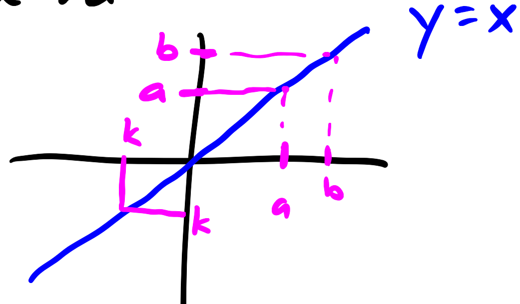
⑤ $$\lim_{x\to a}[f(x)]^n = \left(\lim_{x\to a} f(x)\right)^n$$ n positive integer

$$\left(\lim_{x\to a}[f(x)]^2 = \lim_{x\to a} f(x)f(x) \overset{③}{=} \left(\lim_{x\to a} f(x)\right)\left(\lim_{x\to a} f(x)\right) = \left(\lim_{x\to a} f(x)\right)^2\right.$$

⑥ $$\lim_{x\to a} c = c$$

$$\left(\lim_{x\to 3} 5 = 5\right)$$

⑦ $$\lim_{x\to a} \underline{x} = a$$



⑧ $$\lim_{x \to a} x^n \overset{5}{=} a^n$$

⑨ $$\lim_{x \to a} x^{1/n} = a^{1/n}$$ n positive integer

⑩ $$\lim_{x \to a} \left[f(x)\right]^{1/n} = \left(\lim_{x \to a} f(x)\right)^{1/n}$$